A Possible Modified Dispersion Relation

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Abstract

In this paper we introduced a parameter n to characterize the variation of the speed of light between different inertial systems. In order to satisfy the well-known fundamental principle and not violate some reliable experiments' results, we should impose some necessary constraints on n. Firstly and importantly, the introduction of n should be in agree with the following three principles: (1)we can define the time in the whole space with a prescribed clock synchronization, (2)the time-space is uniform and the space is isotropic and (3)all the inertial systems are equivalent, which are the inheritance of the special relativity (SR). With some constraints on n, we construct a general coordinate transformation to meet the symmetry of inertial systems.

In recent years, many theories have shown the interest in the breakdown of the Lorentz invariance at ultrahigh energy scale, such as the quantum gravity, which imply that the energy of particle has a limited value (called the "Planck energy") rather than be infinite derived from the Lorentz model. So we construct an expression for n to characterize the violation of Lorentz model. And further, by comparing with the well-known rainbow model, we found that the "maximum energy" derived in our paper is somewhat related to the "maximum energy" assumed in the rainbow model.

Keywords

variable speed of light; Lorentz model; ultrahigh energy; rainbow model

1. Introduction

It is well known that the Special Relativity and General Relativity, implying the Lorentz invariance, have already made great achievements, but at the same time the Lorentz violating models are also of some astrophysical interest. In the past few decades the scientific community has shown an intense interest in the theories that contained and investigated the breakdown of Lorentz symmetry in many scenarios [1-5] and also the so-called Deformed Special Relativities (DSR) [6]. For example, a common feature of semi-classical approaches to quantum gravity is the violation of Lorentz invariance due to a deviation from the usual relativistic dispersion relation caused by a redefinition of the physical momentum and physical energy at the Planck scale. And one of the intriguing predictions among various quantum gravity theories, such as loop quantum gravity, non-commutative geometry, and string theories, is the existence of a maximum measurable energy that nears the Planck energy.

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The introduction of "Planck energy or Planck length" in the usual relativistic dispersion relation is considered to be an effective method of combining the relativity and quantum mechanics. Therefor, in the double special relativity (DSR), the "Planck energy or Planck length" is even simply assumed to be a second constant between inertial frames besides the speed of light, and thus the particle's energy-momentum dispersion relation is modified as the following form (set $\boldsymbol{c}$=1) [1,7,8]

$$[1+\chi_1(\frac{E}{E_{LV}})^1+\chi_2(\frac{E}{E_{LV}})^2+O(\frac{E}{E_{LV}})^3]E^2-\boldsymbol{p}^2=m_0^2 \tag{1}$$

where E denotes the total energy of the particle, m_0 denotes the rest mass, $\boldsymbol{p}$ is the momentum, and E_{LV} denotes the energy scale at which Lorentz violating effects become strong, the couplings χ_s (s=1, 2) are determined by the dynamical framework being studied.

Eq. (1) is also called the "rainbow model", which means the theory indicates that the space-time background depends on the energy of a test particle. Due to the large scale of E_{LV}, the variation of c is extremely small at low energy scale that it is very difficult to measure by ordinary experiments. But a feasible approach to solve this problem is to detect photons from the astrophysical objects, such as the Gamma Ray Burst (GRB) events. In recent years, many physicists have used the rainbow model to study the variation of the speed of light in GRB events [9-16]. For example, with the first-order approximation of Eq.(1), Xu [15,16] analyzed the GRB 160509A event. They claimed that there exists a linear relation between the variable speed of light and the photon's energy, and $E_{\mathrm{LV}}\approx 3.6\times 10^{17}$ GeV was obtained.

The idea of rainbow model provide a new approach to explore the violation of Lorentz model, and it inspired us that there may be another relationship between the speed of light and the inertial systems. And in this paper we just try to discuss one possible relationship between the speed of light and the inertial systems, that is, in Sect. 2, we introduced a parameter n to characterize the variation of the speed of light between different inertial systems, and with the constraints from some fundamental principles, we construct a general coordinate transformation between inertial systems to meet the symmetry of inertial systems. In Sect. 3, we construct an expression for n to make the energy of particle have a limit value rather than be infinite derived from the Lorentz model, and the idea is similar to the rainbow model. In Sect. 4, we discussed the relationship between the two "maximum energy", i.e., the "maximum energy" assumed in the rainbow model and the "maximum energy" derived from this paper. In Sect. 5 we summarized the paper.

2. Variable speed of light

It is well known that the above rainbow model presents that the speed of light maybe associated with the photon's energy [15,16], then here we proposed a general hypothesis that: For a light source in vacuum, when it moves at a velocity $\boldsymbol{v}$ relative to an observer in vacuum, then the observed (by the observer) speed of light emitted by the light source is $\boldsymbol{nc}$, where n is a dimensionless quantity, $\boldsymbol{c}$ is the speed of light in vacuum. Obviously, in order not to violate some fundamental principles and experiments' results, we should impose some constraints on the parameter n as follows

1. Firstly, as stated in Einstein's special relativity, using the speed of light it

should be possible to define the (proper) time (note that the only time that makes sense in special relativity is the proper time measured by the clocks carried by physical observers) in the whole space with a prescribed clock synchronization. That is, for a specific inertial system, using the speed of light emitted by a light source that fixed in the specific inertial system, we can calibrate the clock fixed in the inertial system to synchronize. So it requires that

$$n(v = 0, \boldsymbol{c}) = 1 \tag{2}$$

2. Secondly, according to the general concept of time and space that the time-space is uniform and the space is isotropic, it requires that $n(\boldsymbol{v},\boldsymbol{c})$ is independent of the direction of vector $\boldsymbol{v}$ and $\boldsymbol{c}$, namely,

$$n(v,c) = n(-v,c) = n(v,-c) = n(-v,-c) \tag{3}$$

3. In addition, we also follow the principle that all the inertial systems are equivalent.

Based on the above assumptions on the speed of light, next we will derive the coordinate transformation between the two inertial systems $S(\boldsymbol{x},\boldsymbol{y},z,t)$ and $S'(\boldsymbol{x}',\boldsymbol{y}',z',t')$. And here we assume that S' is moving at a velocity $\boldsymbol{v}$ relative to S.

Firstly, for simplicity, we assume that the three spatial coordinates of the two coordinate systems are parallel to each other, and the direction of $\boldsymbol{v}$ is along the $\boldsymbol{x}$-axis or $\boldsymbol{x}'$-axis, which derives that $\boldsymbol{y}=\boldsymbol{y}'$, $\boldsymbol{z}=\boldsymbol{z}'$.

Secondly, since the time-space is uniform, the coordinate transformation between S and S' should be in a linear form (note that in the rainbow model, the coordinate transformation between S and S' is assumed to be in a nonlinear form), and we assume that

$$\boldsymbol{x} = \gamma(\boldsymbol{x}' + \boldsymbol{v}t') \tag{4}$$

Where $\gamma=\gamma(\boldsymbol{v},\boldsymbol{c})$ is a proportionality factor.

The form of Eq. (4) should be invariant under the time reversal symmetry operation. Thus we have

$$\gamma(\boldsymbol{v},\boldsymbol{c}) = \gamma(-\boldsymbol{v},-\boldsymbol{c}) \tag{5}$$

Note that the above we didn't distinguish the direction of vector $\boldsymbol{v}$ and $\boldsymbol{c}$, i.e., the direction of vector $\boldsymbol{v}$ and $\boldsymbol{c}$ maybe along the positive $\boldsymbol{x}$-axis (or $\boldsymbol{x}'$-axis) or along the negative $\boldsymbol{x}$-axis (or $\boldsymbol{x}'$-axis). And if we distinguish the direction of vector $\boldsymbol{v}$ and $\boldsymbol{c}$ by the positive and negative signs, we will obtain four different combinations, i.e., (v, c), $(v, -c)$, $(-v, c)$, $(-v, -c)$. Based on Eq. (5) we can obtain $\gamma(v, c)=\gamma(-v, -c)$, $\gamma(-v, c)=\gamma(v, -c)$.

Now we will solve the expression for γ. If the light signal is emitted by the light source at the moment that the origin of S and S' are coincides, then based on the above assumption on the speed of light, we will obtain

$$\begin{cases} \boldsymbol{x}^2+\boldsymbol{y}^2+\boldsymbol{z}^2=(\boldsymbol{c}t)^2 \\ \boldsymbol{x}'^2+\boldsymbol{y}'^2+\boldsymbol{z}'^2=(n\boldsymbol{c}t')^2 \\ \boldsymbol{y}=\boldsymbol{y}'=0 \\ \boldsymbol{z}=\boldsymbol{z}'=0 \end{cases} \quad \text{and} \quad \begin{cases} \boldsymbol{x}'^2+\boldsymbol{y}'^2+\boldsymbol{z}'^2=(\boldsymbol{c}t')^2 \\ \boldsymbol{x}^2+\boldsymbol{y}^2+\boldsymbol{z}^2=(n\boldsymbol{c}t)^2 \\ \boldsymbol{y}=\boldsymbol{y}'=0 \\ \boldsymbol{z}=\boldsymbol{z}'=0 \end{cases} \tag{6}$$

The first formula of Eq. (6) represents that when the light source is fixed in *S*, then for the observer in *S,* the observed speed of light is $\boldsymbol{c}$ (based on Eq. (2)), while for another observer in *S*', the observed speed of light is $n\boldsymbol{c}$.

Similarly, since *S* and *S*' are equivalent, when the light source is fixed in *S*', then for the observer in *S*', the observed speed of light is $\boldsymbol{c}$, while for another observer in *S,* the observed speed of light is $n\boldsymbol{c}$, which case corresponds to the second formula of Eq. (6).

Based on Eqs. (4)~ (6) and with some simple calculations, it is easy to obtain the coordinate transformation between *S* and *S′*

$$\begin{cases} \boldsymbol{x}'=\gamma(\boldsymbol{x}-\boldsymbol{v}t) \\ t'=\gamma\left(t-\dfrac{\boldsymbol{v}}{k^2(\boldsymbol{v},\boldsymbol{c})}\boldsymbol{x}\right) \end{cases} \tag{7}$$

where $\gamma(\boldsymbol{v},\boldsymbol{c})=1/\sqrt{1-\boldsymbol{v}^2/k^2}$, $k(\boldsymbol{v},\boldsymbol{c})=\boldsymbol{c}\sqrt{n\boldsymbol{v}/(n\boldsymbol{c}-\boldsymbol{c}+\boldsymbol{v})}$.

From Eq. (7) it can be seen that $k(v,c)=k(-v,-c)$, $k(-v,c)=k(v,-c)$, and $\gamma(v,c)=\gamma(-v,-c)$, $\gamma(-v,c)=\gamma(v,-c)$, which satisfy Eq. (5).

Here, in order for the readers to better understand the meaning of Eq. (7), we need to reiterate the meaning of the speed of light. Based on Eq. (7) we can obtain

$$\frac{d\boldsymbol{x}'}{dt'}=\frac{d\boldsymbol{x}-\boldsymbol{v}dt}{dt-\dfrac{\boldsymbol{v}}{k^2}d\boldsymbol{x}}=\frac{d\boldsymbol{x}/dt-\boldsymbol{v}}{1-\dfrac{\boldsymbol{v}}{k^2(\boldsymbol{v},\boldsymbol{c})}\dfrac{d\boldsymbol{x}}{dt}}=f(\boldsymbol{v},\boldsymbol{c}) \tag{8}$$

As stated above that the direction of vector $\boldsymbol{v}$ and $\boldsymbol{c}$ maybe along the positive $\boldsymbol{x}$-axis (or $\boldsymbol{x}'$-axis) or along the negative $\boldsymbol{x}$-axis (or $\boldsymbol{x}'$-axis), and if we distinguish the direction of vector $\boldsymbol{v}$ and $\boldsymbol{c}$ by the positive and negative signs, we will obtain four different combinations, i.e., (*v*, *c*), (*v*, -*c*), (-*v*, *c*), (-*v*, -*c*). Then based on Eq. (8), we have

Case 1: Note that the above we assumed that *S′* is moving at a velocity $\boldsymbol{v}$ relative to *S.* When the light source is fixed in *S*, then for the observer in *S*, the observed speed of light is $\boldsymbol{c}$, while for the observer in *S′*, it has

$$\begin{cases} when\dfrac{dx}{dt}=c,\ \dfrac{dx'}{dt'}=f(v,c)=\dfrac{c-v}{1-\dfrac{v}{k^2(v,c)}c}=f(-v,c)=\dfrac{c+v}{1-\dfrac{-v}{k^2(-v,c)}c}=nc \\ when\dfrac{dx}{dt}=-c,\ \dfrac{dx'}{dt'}=f(v,-c)=\dfrac{-c-v}{1-\dfrac{v}{k^2(v,-c)}(-c)}=f(-v,-c)=\dfrac{-c+v}{1-\dfrac{-v}{k^2(-v,-c)}(-c)}=-nc \end{cases}$$

(9)

Case 2: Similarly, since S and S' are equivalent, when the light source is fixed in S', then for the observer in S', the observed speed of light is $\boldsymbol{c}$, while for the observer in S, it has (note that in this case the velocity of S relative to the light source is -$\boldsymbol{v}$)

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{d\boldsymbol{x}'/dt'-(-\boldsymbol{v})}{1-\dfrac{-\boldsymbol{v}}{k^2(-\boldsymbol{v},\boldsymbol{c})}\dfrac{d\boldsymbol{x}'}{dt'}} = f'(\boldsymbol{v},\boldsymbol{c}) \\ when\dfrac{dx'}{dt'}=c,\ \dfrac{dx}{dt}=f'(v,c)=\dfrac{c+v}{1+\dfrac{v}{k^2(-v,c)}c}=f'(-v,c)=\dfrac{c-v}{1+\dfrac{-v}{k^2(v,c)}c}=nc \\ when\dfrac{dx}{dt}=-c,\ \dfrac{dx}{dt}=f'(v,-c)=\dfrac{-c+v}{1+\dfrac{v}{k^2(-v,-c)}(-c)}=f'(-v,-c)=\dfrac{-c-v}{1+\dfrac{-v}{k^2(v,-c)}(-c)}=-nc \end{cases} \tag{10}$$

Eq. (9) and Eq. (10) can be also expressed in a vector form

$$\begin{cases} \text{when } \dfrac{d\boldsymbol{x}}{dt}=\boldsymbol{c},\ \dfrac{d\boldsymbol{x}'}{dt'}=n\boldsymbol{c} \\ \text{when } \dfrac{d\boldsymbol{x}'}{dt'}=\boldsymbol{c},\ \dfrac{d\boldsymbol{x}}{dt}=n\boldsymbol{c} \end{cases} \tag{11}$$

Eq. (11) shows all the meaning of the speed of light, and it also indicates that Eq. (7) is just the solution of Eq. (6).

Obviously, it can be seen that the forms of Eq. (7) are similar to the Lorentz transformation, i.e., replacing c in the Lorentz transformation with k we can obtain Eq. (7). And it is easy to prove that the Maxwell's Equations are also covariant based on Eq. (7).

Based on Eq. (7) we can obtain the time-space metric

$$ds^2 = -k^2dt^2 + d\boldsymbol{x}^2 \tag{12}$$

Correspondingly, the particle's energy-momentum dispersion relation is

$$E^2 = \boldsymbol{p}^2k^2 + E_0^2 \tag{13}$$

Where $E_0=m_0k^2$ denotes the particle's energy, $E=\gamma m_0k^2$ is the total energy of the particle, $\boldsymbol{p}=\gamma m_0\boldsymbol{v}$ denotes the particle's momentum.

3. Particle's "maximum energy"

As we know, in Lorentz model, the particle's energy tends to be infinite when the particle's velocity is close to the speed of light, however, the idea of DSR or the rainbow model introduces a new constant as the energy limit of the particles, ie., the Planck energy. In this paper, as Eq. (7) shown that if $n\equiv 1$ then Eq. (7) returns to the Lorentz model. But here we would like to discuss another interesting case where n is not always equal to 1, as the general Lorentz violating models suggested [1-6].

Inspired by the idea of rainbow model, we found that Eq. (7) has implied that it

is possible that the particle's energy have a limit. That is, based on Eq. (7), the time-space scaling factor is

$$\gamma = \frac{1}{\sqrt{1-\boldsymbol{v}^2/k^2}} = \frac{1}{\sqrt{\frac{1-\boldsymbol{v}/\boldsymbol{c}}{n}\left(n+\frac{\boldsymbol{v}}{\boldsymbol{c}}\right)}} \tag{14}$$

Eq. (14) inspires us that, when $\boldsymbol{v}=\boldsymbol{c}$, if we assume n=0 simultaneously**,** then it is possible that γ does not tend to be infinite. So here we can try to construct an expression for n. Since n has been constrained in Eq. (2) and Eq. (3), that is

$$\begin{cases} n(v=0,\boldsymbol{c})=1 \\ n(v,c)=n(v,-c)=n(-v,c)=n(-v,-c) \\ \lim\limits_{v\to c}\dfrac{1-\boldsymbol{v}/\boldsymbol{c}}{n}=const. \end{cases} \tag{15}$$

Based on Eq. (15), we can only take a few but finite kinds of expressions for n, and the following expression for n is one of them

$$n=\frac{1}{1-Q}\left(1-Q^{1-v^2/c^2}\right) \tag{16}$$

where Q is a constant.

Besides Eq. (16) one may consider another expression for n, but as we know there are many experiments restricting the violation of Lorentz model [18-26], so we should choose an expression for n to not violate the previous experiments' results. Figure 1 shows the n~v curve when taking $Q=(\mathbf{1}/e)^{10^6}$ randomly as an example.

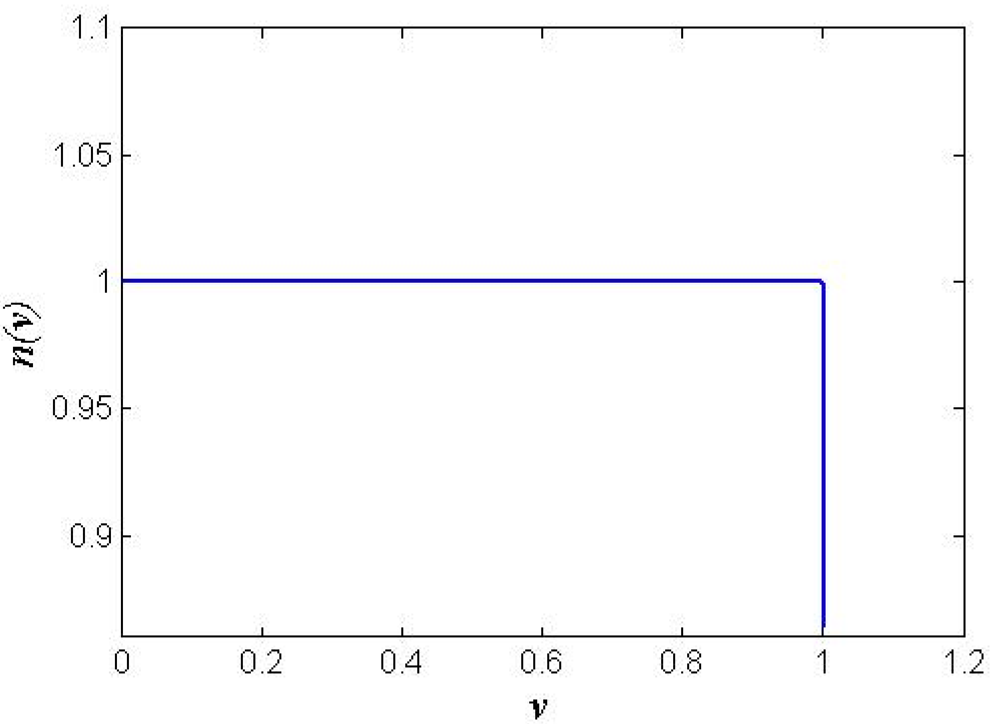

Fig. 1 $n(v)$~v curve when taking $Q=(\mathbf{1}/e)^{10^6}$ (setting c=1)

Thus, based on Eq. (16), the time-space scaling factor limit and the particle's total energy limit are respectively

$$\lim_{v\to c}\gamma=\lim_{v\to c}\frac{1}{\sqrt{1-\boldsymbol{v}^2/k^2}}=\lim_{v\to c}\frac{1}{\sqrt{(\frac{1-\boldsymbol{v}/\boldsymbol{c}}{n})(n+\frac{\boldsymbol{v}}{\boldsymbol{c}})}}=\sqrt{\frac{2\ln Q}{Q-1}}$$

$$\lim_{v\to c}E=\lim_{v\to c}\gamma m_0k^2=E_{QG}=\frac{m_0\boldsymbol{c}^2}{[1-0.5(Q-1)/\ln Q]}\sqrt{\frac{2\ln Q}{Q-1}} \tag{17}$$

4. Comparing with the rainbow model

It can be seen from Fig.1 that the modified particle's energy-momentum dispersion relation will return to the Lorentz case at low or medium energy. So next we will discuss the behavior of particles with ultrahigh energy.

When $\boldsymbol{v}\sim\boldsymbol{c}$ for an ultra-relativistic particle, it can be obtained from Eq. (16) that (setting $\boldsymbol{c}$=1)

$$n=\frac{1}{1-Q}(1-Q^{1-v^2})=\frac{1}{1-Q}[1-Q^{(1+\boldsymbol{v})(1-\boldsymbol{v})}]\approx\frac{1}{1-Q}[1-Q^{2(1-\boldsymbol{v})}]\approx\frac{2\ln Q}{Q-1}(1-\boldsymbol{v}) \tag{18}$$

Then

$$\begin{aligned}\frac{E}{E_{QG}}&=\frac{m_0k^2}{\sqrt{1-\boldsymbol{v}^2/k^2}}/[\frac{m_0\boldsymbol{c}^2}{[1-0.5(Q-1)/\ln Q]}\sqrt{\frac{2\ln Q}{Q-1}}]\\
&=\frac{[1-0.5(Q-1)/\ln Q]}{\sqrt{2\ln Q/(Q-1)}}\frac{n\boldsymbol{v}/(n-1+\boldsymbol{v})}{\sqrt{(1-\boldsymbol{v})(1+\boldsymbol{v}/n)}}\\
&\approx\frac{[1-0.5(Q-1)/\ln Q]}{\sqrt{2\ln Q/(Q-1)}}\frac{2\ln Q/(Q-1)\boldsymbol{v}}{2\ln Q/(Q-1)-1}\frac{1}{\sqrt{1-\boldsymbol{v}+\boldsymbol{v}(Q-1)/(2\ln Q)}}\\
&\approx\frac{[1-0.5(Q-1)/\ln Q]}{\sqrt{2\ln Q/(Q-1)}}\frac{2\ln Q/(Q-1)}{2\ln Q/(Q-1)-1}\frac{1}{\sqrt{(Q-1)/(2\ln Q)+(1-\boldsymbol{v})}}\\
&\approx\frac{1}{\sqrt{2\ln Q/(Q-1)}}[\sqrt{\frac{2\ln Q}{Q-1}}-\frac{1}{2}(\frac{2\ln Q}{Q-1})^{3/2}(1-\boldsymbol{v})]\\
&=1-\frac{\ln Q}{Q-1}(1-\boldsymbol{v})\end{aligned} \tag{19}$$

Based on Eq. (19) we can obtain

$$\frac{\boldsymbol{v}}{\boldsymbol{c}}=1-\frac{Q-1}{\ln Q}+\frac{Q-1}{\ln Q}\frac{E}{E_{QG}} \tag{20}$$

Eq. (20) shows that for an ultra-relativistic particle governed by Eqs. (13) and (16), its velocity is proportional to its energy.

Multiplying $m\boldsymbol{c}^2$ on both sides of Eq. (20), we can obtain

$$\boldsymbol{pc}=m\boldsymbol{c}^2(1-\frac{Q-1}{\ln Q}+\frac{Q-1}{\ln Q}\frac{E}{E_{QG}}) \tag{21}$$

where $\boldsymbol{p}$ is the particle's momentum.

Note that when Q~0, it is easy to prove that $E=mk^2\approx m\boldsymbol{c}^2$ due to $k\approx\boldsymbol{c}$. Then Eq. (21) can be written as

$$\boldsymbol{pc} \approx E(1-\frac{Q-1}{\ln Q}+\frac{Q-1}{\ln Q}\frac{E}{E_{QG}}) \tag{22}$$

On the other hand, in the framework of the DSR or rainbow model, for an ultra-relativistic particle, i.e., $\boldsymbol{v}\sim\boldsymbol{c}$, Eq. (1) can be rewritten as

$$\boldsymbol{pc} \approx E\sqrt{1+\chi_1(\frac{E}{E_{LV}})^1+\chi_2(\frac{E}{E_{LV}})^2+O(\frac{E}{E_{LV}})^3} \tag{23}$$

Comparing Eq. (22) and Eq. (23), we can obtain that

$$\begin{cases} \chi_1\dfrac{1}{E_{LV}}=2(1-\dfrac{Q-1}{\ln Q})(\dfrac{Q-1}{\ln Q})\dfrac{1}{E_{QG}} \\ \chi_2\dfrac{1}{E_{LV}{}^2}=(\dfrac{Q-1}{\ln Q})^2\dfrac{1}{E_{QG}{}^2} \end{cases} \tag{24}$$

Eq. (24) shows that with Eq. (16), the disperse relation in this paper, i.e., Eq. (13), is deeply associate with the disperse relation in rainbow model. But the two "maximum energy", i.e., E_{LV} and E_{QG}, are fundamentally different. That is, E_{LV} is independent of the particle's rest mass, while E_{QG} is depend on the particle's rest mass, which means that different particles have different "maximum energy" .

Further, based on Eq. (24) we can obtain that

$$\frac{\chi_1{}^2}{\chi_2}=4(1-\frac{Q-1}{\ln Q})^2 \tag{25}$$

5. Discussions and Conclusions

To this day, physicists are still trying to use ultrahigh energy events to test the Lorenz model, and an important sign for the violation of Lorenz model is that the local speed of light is variable. In this paper we present a parameter n to characterize the violation of Lorentz model. And given that a typical application of the Lorenz violation model is the quantum gravity, in this paper we try to construct an expression for n to make the particle's energy have a limit. Therefor, we can finally test the value of Q to check the energy scale of violation of the Lorenz model (if $Q\equiv0$ then $n\equiv1$, Eq. (7) returns to the Lorentz case).

But with regret, it seems that the existing experiments still cannot specific the value of Q. The key point is that the value of Q needs to be obtained in the massive particle's experiments, while it is currently difficult to increase the energy of massive particles to be a larger scale due to the limit of technical means. As a general rule, what we can still do is we can obtained the upper bound of Q. For example, based on T. Alvager's experiments [28], it is easy to obtain that $Q<(1/e)^{10^6}$. In addition, Eq. (24) or Eq. (25) can also help us obtain the upper bound of Q indirectly in some experiments aiming to test the rainbow model.

However, the important idea of this paper is introducing a new parameter to characterize the violation of Lorentz model and presenting a possible modified

dispersion relation. Compared with the rainbow model, the model in this paper is more concise in form, that is, there is only one undecided parameter (the value of Q) involved in the model in this paper, while Eq. (1) has at least three parameters to be decided. Although Eq. (24) shows the relationship between the two models, the two models are not equivalent.

Obviously, if Q is not equal to 0, then it will affect the black hole model derived from the General Relativity. We intend to continue this research in the next paper.

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